COMMUTATIVE ALGEBRA
EXERCISE SHEET 9

PROF. DANIEL SKODLERACK

Problem 1 (10 points)**.** Let R be a ring. We call the intersection of all maximal ideals of R the radical of R and denote it by rad(R). Let $\mathfrak{a}$ be an ideal of R contained in the radical of R and M be a finitely generated R-module satisfying $\mathfrak{a}\mathrm{M} = \mathrm{M}$. Prove that M is the zero module.

Problem 2 (10 points)**.** Let R be a ring and $\mathfrak{m}$ be a maximal ideal of R. We consider the localization $\mathrm{R}_{\mathfrak{m}}$. Show that for every positive integer n we have

$$\mathfrak{m}^n/\mathfrak{m}^{n+1} \simeq (\mathfrak{m}_{\mathfrak{m}})^n/(\mathfrak{m}_{\mathfrak{m}})^{n+1}$$

as $\mathrm{R}/\mathfrak{m}$-modules.

Problem 3 (10 points)**.** Consider the cuspidal cubic

$$\mathrm{C} = \mathrm{V}(X^3 - Y^2)$$

in $\mathbb{C}^2$. Compute for every point $\mathrm{P} \in \mathrm{C}$ the $\mathbb{C}$-dimension of $\mathfrak{m}_{\mathrm{P}}/(\mathfrak{m}_{\mathrm{P}})^2$ where $\mathfrak{m}_{\mathrm{P}}$ is the vanishing ideal of P.

Problem 4 (10 points)**.** Prove the Snake lemma: Lemma 123.

Date: Please hand in before the lecture by **16.11.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.