COMMUTATIVE ALGEBRA
EXERCISE SHEET 8

PROF. DANIEL SKODLERACK

Problem 1 (10 points)**.** Prove Lemma 105(2).

Problem 2 (10 points)**.** Let R be a ring. Prove that an R-module M is R-flat if and only if all localizations $M_{\mathfrak{p}}$, $\mathfrak{p} \in \mathrm{Spec}(R)$, are $R_{\mathfrak{p}}$-flat. (Meant is the localization with at the set $R \backslash \mathfrak{p}$.)

Problem 3 (10 points)**.** Prove that every projetive module is flat.

Problem 4 (10 points)**.** Let S_1 and S_2 be multiplicatively closed subsets of R. Then

$$S_1 S_2 := \{s_1 s_2 \mid \ s_1 \in S_1, \ s_2 \in S_2\}$$

is a multiplicatively closed set. Assume $0 \notin S_1 S_2$. show that

$$R_{S_1 S_2} \simeq (R_{S_1})_{\phi(S_2)}$$

where ϕ is the canonical map from R to R_{S_1}.

Date: Please hand in before the lecture by **09.11.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.