

**COMMUTATIVE ALGEBRA**  
**EXERCISE SHEET 7**

PROF. DANIEL SKODLERACK

**Problem 1** (10 points). Prove Proposition 93(2).

**Problem 2** (10 points). Finish the proof of Proposition 93(3).

**Problem 3** (10 points). Prove that  $(\mathbb{Z}/n\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z}/m\mathbb{Z})$  is isomorphic to  $\mathbb{Z}/(\gcd(m, n)\mathbb{Z})$  as rings for  $n, m \in \mathbb{N}$ .

**Problem 4** (10 points (direct products and tensor product)). Let  $M_i$ ,  $i \in I$ , be a family of  $R$ -modules and let  $N$  be an  $R$ -module. Is there a counterexample for

$$\prod_{i \in I} (M_i \otimes_R N) \cong \left( \prod_{i \in I} M_i \right) \otimes_R N.$$

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*Date:* Please hand in before the lecture by **02.11.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.