COMMUTATIVE ALGEBRA
EXERCISE SHEET 6

PROF. DANIEL SKODLERACK

Problem 1 (10 points)**.** Let M be an R-module which satisfies $(inj+)_{\mathrm{R}}$ and let N be a direct summand of M. Show that N satisfies $(inj+)_{\mathrm{R}}$. You are not allowed to use Theorem 84, because its proof uses this result.

Problem 2 (10 points)**.** Prove that the ring

$$\begin{pmatrix} \mathbb{Z} & \mathbb{Z} \\ 0 & \mathbb{Z} \end{pmatrix}$$

is not left hereditary.

Problem 3 (10 points)**.** Prove Theorem 75.

Problem 4 (10 points)**.** Let N be an R-module and Q be an abelian group. Show that $\mathrm{M} := \mathrm{Hom}_{\mathbb{Z}}(\mathrm{N}, \mathrm{Q})$ is an R-module with respect to the structure

$$(r.\phi)(n) := \phi(rn),\ \phi \in \mathrm{M},\ r \in \mathrm{R},\ n \in \mathrm{N}.$$

Date: Please hand in before the lecture by **26.10.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.