COMMUTATIVE ALGEBRA
EXERCISE SHEET 5

PROF. DANIEL SKODLERACK

Problem 1 (10 points, inj+)**.** Show that $(inj+)_{\mathrm{R}}$ implies $(inj)_{\mathrm{R}}$. We recall that $(inj)_{\mathrm{R}}$ is just the injectivity property Definition 54(Part 2).

Problem 2 (10 points, divisible modules)**.** Let R be a principal ideal domain. Then every R-divisible module is R-injective.

Problem 3 (10 points, not projective modules)**.** Show that the ideal $(2., X)_{\mathbb{Z}[X]}$ is not a projective $\mathbb{Z}[X]$-module.

Problem 4 (10 points, localization and injectiveness.)**.** Let R be an integral domain and F its field of fractions. Then F is an R-module via the inclusion. Show that F is an injective R-module.

Date: Please hand in before the lecture by **19.10.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.