COMMUTATIVE ALGEBRA
EXERCISE SHEET 3

PROF. DANIEL SKODLERACK

Problem 1 (10 points)**.** Compute $\mathrm{End}_{\mathbb{Z}}(\mathbb{Q}/\mathbb{Z})$.

Problem 2 (10 points, equivalent criterion for a short exact sequence to be split)**.** Let $0 \to \mathrm{M}_1 \xrightarrow{\alpha} \mathrm{M}_2 \xrightarrow{\beta} \mathrm{M}_3 \to 0$ be a short exact sequence of R-modules. Then are equivalent:

(i) The sequence is split.
(ii) There is an R-module homomorphism $t : \mathrm{M}_2 \to \mathrm{M}_1$ such that $t \circ \alpha = \mathrm{id}_{\mathrm{M}_1}$.

Problem 3 (10 points, full lattices)**.** Let W be an n-dimensional $\mathbb{Q}$-vector space ($n \geqslant 1$) and let Λ be a finitely generated $\mathbb{Z}$-submodule of W. Then are equivalent:

(i) Λ is $\mathbb{Z}$-module isomorphic to $\mathbb{Z}^n$
(ii) Λ contains a $\mathbb{Q}$-basis of W.
(iii) Λ spans W over $\mathbb{Q}$.

Problem 4 (10 points, noetherian modules)**.** Let $0 \to \mathrm{M}_1 \xrightarrow{\alpha} \mathrm{M}_2 \xrightarrow{\beta} \mathrm{M}_3 \to 0$ be a short exact sequence of R-modules. Suppose that M_1 and M_3 are noetherian. Show that M_2 is noetherian.

Date: Please hand in before the lecture by **09.10.2021**. For all exercises the results need to be proven. You are allowed to use results from the Abstract Algebra course.