ABSTRACT ALGEBRA
EXERCISE SHEET 14

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Problem 1 (10 points)**.** Show that the field extension $\mathbb{F}_3(\mathrm{T},\mathrm{S})|\mathbb{F}_3(\mathrm{X},\mathrm{Y})$ with $\mathrm{T}^3 = \mathrm{X}$ and $\mathrm{S}^3 = \mathrm{Y}$ is not primitive, i.e. there does not exists an element $\alpha \in \mathbb{F}_3(\mathrm{T},\mathrm{S})$ such that

$$\mathbb{F}_3(\mathrm{T},\mathrm{S}) = \mathbb{F}_3(\mathrm{X},\mathrm{Y})[\alpha]$$

Problem 2 (10 points)**.** Find the splitting field E of $\mathrm{X}^7 - 9$ over $\mathbb{Q}$ in $\mathbb{C}$ and find an intermediate field E_1 different from E and $\mathbb{Q}$ such that such that $\mathrm{E}|\mathrm{E}_1$ and $\mathrm{E}_1|\mathbb{Q}$ are normal.

Problem 3 (10 points)**.** Let E|F be an algebraic extension and $\bar{\mathrm{E}}$ be an algebraic closure of E. The normal hull of E|F in $\bar{\mathrm{E}}$ is the intersection of all fields L which satisfy $\bar{\mathrm{E}} \supseteq \mathrm{L} \supseteq \mathrm{E}$ and L|F is normal. Compute the normal hull of $\mathbb{Q}[\sqrt{3}, 3^{\frac{1}{5}}]|\mathbb{Q}$.

Problem 4 (10 points)**.** Let $\mathrm{E}_1|\mathrm{F}$ and $\mathrm{E}_2|\mathrm{F}$ be two algebraic field extensions in some algebraic closure $\bar{\mathrm{F}}$ of F. Suppose $\mathrm{E}_1 \cap \mathrm{E}_2$ is equal to F and $\mathrm{E}_1|\mathrm{F}$ is Galois. Prove $\mathrm{E}_1|\mathrm{F}$ and $\mathrm{E}_1\mathrm{E}_2|\mathrm{E}_2$ have the same degree.

Date: Please hand in before the lecture by **03.06.2021**. For all exercises the results need to be proven.