ABSTRACT ALGEBRA
EXERCISE SHEET 11

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Problem 1 (10 points)**.** Let E|F be a field extension and S be a finite subset of E. Prove that $\mathrm{F}(S)|\mathrm{F}$ is algebraic if and only if $\mathrm{F}[S] = \mathrm{F}(S)$. Does the statement still hold if we allow S to be infinite?

Problem 2 (10 points)**.** Show that there is a countable algebraically closed field of characteristic zero. The characteristic of a field F is defined as follows:

$$\mathrm{char}(\mathrm{F}) := \begin{cases} \min\{n \in \mathbb{N} \mid n1_{\mathrm{F}} = 0_{\mathrm{F}}\}, & \text{if there exists an } n \in \mathbb{N} \text{ such that } n1_{\mathrm{F}} = 0_{\mathrm{F}} \\ 0, & \text{if for all } n \in \mathbb{N} \text{ we have } n1_{\mathrm{F}} \neq 0_{\mathrm{F}} \end{cases}.$$

Problem 3 (10 points)**.** Let E|F be a field extension and let α, β be elements of E which are algebraic over F. Show that $\alpha + \beta, \alpha - \beta, \frac{\alpha}{\beta}$ (if $\beta \neq 0$) are algebraic over F.

Problem 4 (10 points)**.** Let E|F be an algebraic field extension and L be an algebraically closed field containing F. Show that there is a field homomorphism φ from E to L whose restriction to F is the identity of F.

Date: Please hand in before the lecture by **13.05.2021**. For all exercises the results need to be proven.