ABSTRACT ALGEBRA
EXERCISE SHEET 9

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Problem 1 (10 points)**.** We will later prove in the lecture that $\mathbb{Z}[i]$ is a. factorial ring. For this exercise you are allowed to use this fact. Let p be a prime number congruent to 1 modulo 4.

(i) Show that there is an element x of $\mathbb{Z}$ such that p divides $x^2 + 1$.
(ii) Use the fact, that $\mathbb{Z}[i]$ is factorial, to prove that p is in $\mathbb{Z}$ equal to a sum of two squares.

Problem 2 (10 points)**.** Find a prime factorization of the following polynomials in $\mathbb{Z}[X, Y]$.

(i) $20X^3 + 42X^2 + 48X + 45$
(ii) $X^5 - X^3Y^2 - X^2Y^2 + Y^4$

Problem 3 (10 points)**.** (i) Show that $A[X_1, X_2, ...]$ is a UFD if A is a UFD.

(ii) Show that there is an integral domain which contains non-zero non-units which are not products of finitely many irreducible elements.

Problem 4 (10 points)**.** Which of the elements 7 and 13 is

(i) irreducible
(ii) a prime element

in $\mathbb{Z}[\sqrt{(}-5)]$?

Date: Please hand in before the lecture by **29.04.2021**. For all exercises the results need to be proven.