

ABSTRACT ALGEBRA
EXERCISE SHEET 7

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Problem 1 (10 points). Let F be a finite field. Then $|F|$ is a prime power.

Problem 2 (10+10 points). (i) Let G be a group and let g_1, g_2 be commuting elements of G of co-prime finite order. Show that the order of g_1g_2 is equal to product of the order of g_1 and the order of g_2 .

(ii) Let F be a finite field. Show that F^\times is cyclic.

Problem 3 (10 points). Find all solutions for the following congruence systems. (All congruences need to be satisfied.)

(i) $P(X, Y) \equiv_{(X, Y-1)} 1$, $P(X, Y) \equiv_{(Y^2)} XY$, $P(X, Y) \in \mathbb{R}[X, Y]$.

(ii) $z \equiv_7 3$, $z \equiv_{16} 4$, $z \equiv_{10} 2$, $z \in \mathbb{Z}$.

Problem 4 (10 points*). Find up to isomprphy all non-abelian groups of order 8. (This question was part of the quiz, but I think it is a good idea to fill in all the details nicely.)